Neutrinos: Harbingers of New Theories *

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Dedicated to E. C. George Sudarshan on the occasion of his sixtieth birthday

We catalog models with massive neutrinos, emphasizing their uses as probes of physics beyond the standard model. We discuss their experimental implications, in terms of neutrino oscillations and the MSW effect.

1. Introduction

At first, the discovery of parity violation in the Weak Interactions seemed to greatly complicate the formulation of the effective four-fermi interaction which causes β decays and other Weak Interactions. Further the data was often contradictory; not only did it not specify and particular universal form (although it favored the wrong one), nor did it point to the existence of a universal interaction responsible for *all* the Weak Interactions. Simplicity was regained by a careful consideration of the nature of the antineutrinos emitted in β decay. By invoking chirality, and disregarding some results fom experiment, Sudarshan was led to the correct formulation of the Universal Interaction, according to which only right-handed antineutrinos are emitted in β decay [1]. Thus neutrinos provided the basic hint which eventually led to the formulation of the remarkably successful Standard Model.

Today, some thirty four years later, neutrinos are again leading theorists to new frontiers. This time it is neutrinos from the Sun which are providing tantalizing hints on the structure of Interactions beyond the Standard Model. In honoring George Sudarshan on this occasion, it is only fitting to acknowledge his seminal contribution by discussing some aspects of the present situation in neutrino physics. Just as it was before the V–A theory, today's experimental situation is confused, so that theorists once again look for the simplicity and elegance of the neutrinos to lead them towards the formulation of a more cohesive picture of the fundamental interactions.

In the following, we describe neutrinos, their masses and electromagnetic moments in general terms, and review arguments for their apparent masslessness, with emphasis on the role of the various lepton numbers. We discuss the ways in which neutrino masses can be incorporated in the Standard Model, distinguishing between models with new fermions and models with new lepton number carrying Higgs. Vacuum oscillations, one of the consequences of massive neutrinos, are then discussed, followed by a presentation on solar neutrinos, their expected detection, and a summary of the present status of experimental searches. Oscillations in the Sun, and their experimental signature are reviewed.

2. Neutrinos in the Standard Model

Neutrinos produced in β decay appear as left-handed particles or right-handed antiparticles. No right handed neutrinos have been observed. Accordingly, their mathematical description is the simplest. Neutrinos are defined by local fields which transform as spinor representations of the Lorentz group, gener-

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ated by the three rotations J_i and the three boosts K_i, is $SO(3, 1)$. The non-unitary combinations $J_i + iK_i$ and $J_i - iK_i$ generate two commuting SU_2's, which are related either by conjugation $C(i \to -i)$ or by parity $P(J_i \to J_i, K_i \to -K_i)$, so that both sustain the combined operation of CP. The left-handed neutrino fields, $\nu_L^a(x)$, where a denotes their flavor, transform non-unitarily as the **(2, 1)** representation. Conversely, a right handed field $N_R(x)$ transforms as the **(1, 2)** representation. The fields $\bar{\nu}_L^a(x) = \sigma_2 \nu_L^{a*}(x)$ transform as right-handed spinors (* means complex conjugation), which enables us to make a bilinear that transforms as a current i.e. like **(2, 2)**

$$J^{\mu ab}(x) = \nu_L^{a\dagger}(x)\,\sigma^\mu \nu_L^b(x)\,,$$

where $\sigma^\mu = (\sigma^0 \equiv 0, \boldsymbol{\sigma})$ denote the 2×2 unit and Pauli matrices, respectively. The current we have just written above is the neutrino part of the neutronal current when summed over the flavors. In the following, we will dispense with the L and R subscripts: a spinor field without a bar over it is understood to transform as **(2, 1)**, one with a bar over it as **(1, 2)**.

One can construct other bilinears in neutrino fields. From the SU(2) property that $\mathbf{2} \otimes \mathbf{2} = \mathbf{1}_A \oplus \mathbf{3}_S$, it is easy to arrive at the combinations

$$\nu^{aT}(x)\,\sigma^2 \nu^b(x) \quad \text{and} \quad \nu^{aT}(x)\,\sigma^2 \boldsymbol{\sigma}\,\nu^b(x)\,,$$

the first corresponding to the Lorentz scalar **(1, 1)**, the second to part of the antisymmetric tensor **(3, 1)**. They have different flavor symmetry properties. Since the $\nu^a(x)$ obey Fermi statistics, the scalar combination is symmetric in the flavor indices a, b because of antisymmetrization on the Lorentz indices. This combination is of course the Lorentz invariant Majorana mass [2]; it can appear in the Lagrange density as

$$\mathscr{L}_{\text{Maj}} = m_{(ab)}\,\nu^{aT} \sigma^2 \nu^b + \text{conjugate}\,,$$

where we have indicated the flavor symmetrization by round brackets. If each field ν^a is assigned a global lepton number L_a, this mass term violates it by two units. For a charged particle such as the electron, described by both left and right handed fields $e_L(x)$ and $e_R(x)$, the usual mass term is written as

$$\mathscr{L}_{\text{Dirac}} = m\,e_L^\dagger e_R + \text{conjugate}\,;$$

and it is seen to converse lepton number. We call this type of mass a Dirac mass to distinguish it from the Majorana mass which violates lepton number by two units. To describe both types of masses in the same formalism, we need two left-handed fields $N_L^a(x)$ $a = 1, 2$. Their mass terms can be written as

$$\mathscr{L} = (N_L^{1T} N_L^{2T})\,\sigma_2 \begin{pmatrix} m_1 & m \\ m & m_2 \end{pmatrix} \begin{pmatrix} N_L^1 \\ N_L^2 \end{pmatrix} + \text{conjugate}\,.$$

The different entries in the 2×2 matrix are labeled in terms of N-number: $m_{1,2}$ violate $N_{1,2}$-number by two units, while preserving $N_{2,1}$-number, and the off-diagonal term m violates both N_1 and N_2-number while preserving their difference. The off-diagonal term of the (Majorana) mass matrix, if we identify $N_L^2 \equiv \sigma^2 N_R^*$ and $N_L^1 \equiv N_L$, simply becomes a Dirac mass. For charged particles this formalism is not really necessary, but for neutral fermions there is no local quantum number which forbids the diagonal elements, and this formalism is necessary, with the general mass matrix as a complex, symmetric matrix, of the form

$$\begin{pmatrix} \text{Maj} & & \text{Dirac} \\ & \text{Maj} & \\ \text{Dirac} & & \text{Maj} \end{pmatrix}.$$

Massive neutrinos can be of two types. Either the mass is Majorana and violates lepton number, and there need not be extra fermion degrees of freedom, or it is Dirac and preserves lepton number at the price of introducing for each massive neutrino a right-handed partner with the same lepton number.

Let us now turn to the second bilinear construct, with the Lorentz quantum numbers of electromagnetic moments. Recall that an antisymmetric tensor $F_{\mu\nu}$ can be decomposed into two three vector $E_i = F_{0i}$ and $B_i = \frac{1}{2}\varepsilon_{ijk} F_{jk}$, and these two vectors can be arranged as a complex (Hertz) vector $\boldsymbol{E} \pm i\boldsymbol{B}$, corresponding to **(3, 1)** and **(1, 3)**, respectively. Alternatively one can view the **(3, 1)** as a complex antisymmetric tensor which obeys the complex self-duality condition

$$F_{\mu\nu} = \frac{i}{2}\,\varepsilon_{\mu\nu}{}^{\varrho\sigma} F_{\varrho\sigma}\,.$$

Fermi statistics then demands that the flavor indices be antisymmetrized: there is no electromagnetic moment for only one left-handed field – just as for dancing the tango, it takes two. For charged particles, this is well understood since the magnetic moment makes a transition between left- and right-chiralities. The constructs of interest are of the form

$$\nu^{aT}\sigma_2\boldsymbol{\sigma}\nu^b\,,\qquad \bar{\nu}^{aT}\sigma_2\boldsymbol{\sigma}\bar{\nu}^b\,.$$

One can form one particular linear combination even under CP, the other odd, corresponding to the magnetic and electric moments, respectively. In the Lagrange density, these will be multiplied by the field strengths and thus have dimension five, and be, if allowed at all, generated by loop diagrams. These terms also violate the total lepton number by two units.

In the Standard Model, the left-handed neutrinos appear as the upper components of the three flavor isodoublets L_a,

$$L_a=\begin{pmatrix}\nu^a\\ e^a\end{pmatrix}\sim(\mathbf{2},\mathbf{1};\mathbf{2},\mathbf{1}^c)_{-1}\,,\qquad a=1,2,3\,,$$

while the left-handed antielectron transforms as $\bar{e}_a\sim(\mathbf{2},\mathbf{1};\mathbf{2},\mathbf{1}^c)_2$. The notation denotes the representations under $(SU_2\otimes SU_2;\,SU_2^{\mathrm{W}}\otimes SU_3^{\mathrm{C}})_Y$, the first two SU(2) refer to the Lorentz group, Y is the hypercharge, SU_2^{W} is the weak isospin, SU_3^{C} the color group. The charged fermions get a mass through the Yukawa couplings

$$Y_{ab}^{(-1)}L_a^T\sigma_2\bar{e}_bH^*\,,$$

where $Y_{ab}^{(-1)}$ is a complex 3×3 matrix of Yukawa coupling constants, and H is the Higgs doublet $H\sim(\mathbf{1},\mathbf{1};\,\mathbf{2},\mathbf{1}^c)_{-1}$. Upon vacuum breakdown, these translate into Dirac masses for the charged leptons. One can always write the 3×3 flavor matrix in the form

$$Y^{(-1)}=U_e^TD_eV_e\,,$$

where D_e is a diagonal matrix, and U_e and V_e are unitary matrices. In the Standard model, these unitary matrices can always be absorbed in a redefinition of the fields, and therefore there are only three couplings in this sector, each proportional to the Dirac masses of the charged leptons.

The Majorana masses, if they were to appear at all, would be generated by Lorentz invariant constructs of the form $L_a^T\ldots\sigma_2L_b$, where the dots stand for the relevant Clebsch-Gordan coefficients. Such bilinears in the lepton doublets have the quantum numbers

$$(\mathbf{1},\mathbf{1};\,\mathbf{3},\mathbf{1}^c)_{-2\,(ab)}\quad\text{and}\quad(\mathbf{1},\mathbf{1};\,\mathbf{1},\mathbf{1}^c)_{-2\,[ab]}\,.$$

The first combination corresponds to symmetrized family indices, the second to antisymmetrized ones. Of these, only the first contains an electrically neutral combination, capable of generating a Majorana mass for the neutrinos. This channel has the quantum numbers of a weak isotriplet. There is independent evidence from the ϱ parameter measurement of the relative strength between neutral and charged currents that electroweak breaking in the isotriplet sector is very small, which is certainly consistent with the fact that neutrino masses, if they exist at all, are very tiny.

The absence of Higgs triplets in the Standard Model means that there are no tree-level masses for the neutrinos. But one can make a triplet out of doublets, and we can expect an effective contribution of dimension five to the Lagrangean which satisfies all the local symmetries of the Standard Model,

$$\frac{m_\nu^{ab}}{v^2}L_a^T\sigma_2\tau_2\boldsymbol{\tau}L_b\cdot H^T\tau_2\boldsymbol{\tau}H\,,$$

where the τ-matrices are the weak isospin generators, and $v=245$ GeV is $G_{\mathrm{F}}^{-1/2}$. It will never be generated in perturbation theory because of lepton number: in the Standard Model, where the Higgs field carries no lepton number, this operator violates lepton number by two units. It is a global conservation law that forbids neutrino masses.

Can this be circumvented? It is well known that because of the weak $SU(2)^{\mathrm{W}}$ non-Abelian anomaly, the electroweak theory violates the left handed part of L, but preserves B−L [3]. This does not affect our conclusions about this operator since it still violates B−L by two units as well. However, a term of the form

$$L_a^T\sigma_2\tau_2\boldsymbol{\tau}L^b\cdot H^T\tau_2\boldsymbol{\tau}H\mathcal{O}\,,$$

where $\mathcal{O}$ is a weak isosinglet with two units of B, could appear in the Lagrangean. If it exists, and acquires a non-zero value in the electroweak vacuum, the tiniest of tiny neutrino masses would be generated. But this is still forbidden by the relative lepton numbers [4]. In other models, for instance in $SU(5)$, the relative lepton numbers are violated and neutrino masses are thought to be zero; this could be interesting if such an $\mathcal{O}$ existed at all. An example of such an operator is a combination of six antiquarks $\mathcal{O}=\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}$. Unfortunately, there is a theorem [5] which states that QCD does not break vectorial symmetries. It is based on the reality of the quark determinant in QCD and is of very general import, although the theorem may be weakened when electroweak Yukawa couplings of the quarks are included.

The discussion of Electromagnetic Moments in the Standard Model parallels the previous one. In terms of the combination

$$W_i \equiv W_{0i} + i\frac{1}{2}\varepsilon_{ijk} W_{jk}$$

for the weak isospin and

$$B_i = B_{0i} + \frac{1}{2}\varepsilon_{ijk} B_{jk}$$

for the weak hypercharge, these invariant operators are of two types – those which are in the weak isotriplet mode and thus antisymmetric in family indices, and those in the weak isosinglet mode which are symmetric in family indices. They are

$$L^T_{[a}\sigma_2\sigma_i\tau L_{b]}\cdot\begin{cases}H^T\tau_2\tau H B_i\\(H^T\tau_2\tau H\times W_i)\end{cases},$$

and

$$L^T_{(a}\sigma_2\sigma_i\tau_2 L_{b)} H^T\tau_2\tau H\cdot W_i\,.$$

With two Higgs doublets, the latter can contribute to the hypercharge moment through

$$L^T_{(a}\sigma_2\sigma_i\tau_2 L_{b)} H^T\tau_2 H' B_i\,.$$

Only the family antisymmetric combinations give rise to electromagnetic moments; none of these operators can be generated in perturbation theory because of lepton number violation.

In the Standard Model, neutrinos cannot get masses. If experiments were to reveal that neutrinos are massive, it would constitute the first evidence of Physics beyond the Standard Model.

3. Standard Model Extensions with Massive Neutrinos

Massive neutrinos would force two types of extensions of the Standard Model. In the first, the neutrino masses are of the Dirac type with preservation of some lepton number; in the second the neutrino masses are of Majorana type with a concomitant breakdown of lepton number. In the first alternative, new fermion degrees of freedom are necessarily introduced to serve as the Dirac partners of the usual neutrinos. In the second, extra fermions need not be introduced.

a) Lepton Number Conserving Extensions

In this case the idea is to provide Dirac partners to the neutrinos produced in the weak interactions. After electroweak breaking these Dirac masses will take the generic form $m\,\nu^\dagger\bar{N}$. At the electroweak level we must specify the classification of the new field(s) $\bar{N}\equiv\sigma_2\sigma^*_{\mathrm{R}}$ under weak isospin. The easiest case is that of weak isosinglets with no hypercharge; because of their lack of weak interactions they are called "sterile" neutrinos.

In general these new fields $N^m(x)$ are labeled by their own flavor index m, $m = 1, \ldots M$; it need not be the same as the usual flavor, although in many GUT-like extensions, and in the following it is taken to be the same. These isosinglet fermions couple to the lepton doublets via the Yukawa interactions

$$Y^{(0)}_{ab}\, L^T_a\sigma_2 N_b\tau_2 H + \text{conjugate}\,,$$

where $Y^{(0)}_{ab}$ is an unknown matrix. At least one global lepton number is preserved, with $\mathrm{L} = -1$ for the N_m's. After the Higgs takes on its vacuum value of 245 GeV, the left handed neutrinos acquire a Dirac mass of the order of $245\times Y^{(0)}$ GeV. Yet, the experimental limits [6] on neutrino masses are very tiny,

$$m_{\nu_e}\lesssim 9\,eV\,;\quad m_{\nu_\mu}\lesssim 0.27\,MeV\,;\quad m_{\nu_\tau}\lesssim 35\,MeV,$$

which means that the $Y^{(0)}$ coupling constants must themselves be very small, in the range $Y^{(0)}\lesssim(10^{-10}-10^{-4})$. If one is willing to accept the presence of such tiny couplings (after all, we already have $m_e = 10^{-6}M_{\mathrm{W}}$!), this represents a viable extension of the Standard Model.

b) Lepton Number Violating Extensions

Majorana mass extensions of the Standard Model imply breakdown of the total lepton number. This means that new degrees of freedom with lepton number must be added, so that breakdown of lepton number may be achieved either by their interactions of masses. Generically, these new fields can either be spinless, i.e. Higgs-like, or fermions. We have just seen how to add to the Standard Model fermionic degrees of freedom with lepton number; we now generalize this discussion.

i) Majorana Masses

Let us take up our previous case. Sterile Dirac partners can have a Majorana mass without violating local electroweak symmetries. The only obstacle is the global total lepton number, which we have agreed to break it anyhow. The electroweak quantum numbers of the Majorana mass matrix, after electroweak breaking, appear as

$$(\nu^T N^T)\sigma_2\begin{pmatrix}\Delta I_w = 1 & \Delta I_w = \frac{1}{2}\\ \Delta I_w = \frac{1}{2} & \Delta I_w = 0\end{pmatrix}\begin{pmatrix}\nu\\ N\end{pmatrix},$$

where the entries of the mass matrix break weak isospin as shown. In the absence of a Higgs triplet, the $\Delta I_w = 1$ entry will be zero at three level. The $\Delta I_w = \frac{1}{2}$ off-diagonal entries originate in the Dirac-type of Yukawa coupling and are of the order of the electroweak scale. On the other hand, the $\Delta I_w = 0$ entry is not constrained by any known physics (except perhaps that it should be smaller than Planck mass!). Here, we take this entry to stem from a bare mass therm in Lagrangean. We write the Majorana mass matrix in the form

$$\mathcal{M} = \begin{pmatrix}0 & v\,Y^{(0)}\\ v\,Y^{(0)} & M\end{pmatrix},$$

where M is a symmetric matrix, which can be diagonalized by a unitary transformation,

$$M = U_0^T D_0 U_0 \quad \text{(Schur's theorem)},$$

and the Yukawa couplings $Y^{(0)}$ matrix can be decomposed as before

$$Y^{(0)} = U_\nu^\dagger D_\nu V_\nu\,, \quad U_\nu,\, V_\nu \text{ unitary}, \quad D_\nu \text{ diagonal}.$$

Since the $\Delta I_w = 0$ scale is not constrained by present experiments, motivated by theoretical prejudices, let us assume it is much larger than the $\Delta I_w = \frac{1}{2}$ electroweak breaking scale. Let $\varepsilon \ll 1$ denote the ratio of the electroweak to $\Delta I_w = 0$ scales,

$$\varepsilon = \frac{\Delta I_w = \frac{1}{2}}{\Delta I_w = 0}.$$

In this approximation [7], we can write the full matrix in the form

$$\begin{aligned}\mathcal{M} &= \mathcal{U}^T \mathcal{D}\, \mathcal{U}\\ &= \begin{pmatrix}U_{11}^T & \varepsilon U_{21}^T\\ \varepsilon U_{12}^T & U_{22}^T\end{pmatrix}\begin{pmatrix}\varepsilon^2 D_\nu & 0\\ 0 & D_0\end{pmatrix}\begin{pmatrix}U_{11} & \varepsilon U_{12}\\ \varepsilon U_{21} & U_{22}\end{pmatrix},\end{aligned}$$

where $\mathcal{U}$ is unitary, so that

$$\begin{aligned}&U_{11} U_{11}^T + \varepsilon^2 U_{12} U_{12}^T = 1\,,\\ &U_{22} U_{22}^T + \varepsilon^2 U_{21} U_{21}^T = 1\,,\\ &U_{11} U_{21}^\dagger + U_{21} U_{22}^\dagger = 0\,.\end{aligned}$$

The charged current density is now written as

$$J_\mu^+ = \varepsilon^\dagger \sigma_\mu \nu = \hat{e}^\dagger U_e (U_{11}^\dagger \hat{\nu} + \varepsilon U_{21}^\dagger \hat{N}),$$

where the hat denotes mass eigenstates. In this case, the M mixing matrix is just

$$U_M = U_e U_{11}^\dagger,$$

and since U_{11} is almost unitary, it can be decomposed *à la* Iwasawa, yielding

$$U_e U_{11}^\dagger = P\, U_M'\, P',$$

where P and P' are diagonal phase matrices. The phase matrix P can be absorbed in $\hat{e}_L^\dagger$ and then transferred into $\hat{e}_R$, but the P' phase matrix cannot be absorbed into $\hat{\nu}$, since it would reappear in the mass term $\varepsilon^2\, \hat{\nu}^T \sigma_2 D_\nu \hat{\nu}$. There are more CP-violating phases in the lepton sector: here we have 3 phases in total, but the only thing that forbids the two extra phases from being absorbed is the Majorana mass matrix, which is $\mathcal{O}(\varepsilon^2)$ – so we expect their experimental consequences to be very tiny.

The mixing matrix U_{11} and the Majorana masses of the light neutrinos $m_\nu = \varepsilon^2 D_\nu$ are determined from

$$Y^{(0)} \frac{v^2}{M} Y^{(0)T} = -\, U_{11}^T \varepsilon^2 D_\nu U_{11}\,.$$

This formula is applicable provided that M, the $\Delta I_w = 0$ entry in the Majorana mass, has no zero eigenvalue. This brings us to the advantage of this mechanism; if $\varepsilon \ll 1$, we see that the Majorana masses of the normal neutrinos appear depressed by $\mathcal{O}(\varepsilon^2)$, so that the smallness of the ν masses would be explained by the ratio of the $\Delta I_w = \frac{1}{2}$ to $\Delta I_w = 0$ scales [8]. We should not that this scenario occurs naturally in the context of GUTS and other popular extensions of the Standard Model. It is therefore a favorite of many theorists – it is very pleasing because the tiny limits on the neutrino masses are linked to the ratio of scales, just like the tiny limits on the proton width in GUTs is linked to the appearance of another scale. This affords the theorists with a uniform view of these different phenomena. For right thinking theorists, the only natural scale is that given by Newton's constant. The mysteries of the Standard Model are the smallness of the electroweak breaking scale in Planck units (24.6 Atto-Plancks!), its near equality (in the astrophysical sense) with the QCD confinement scale, and then the appearance of yet smaller numbers such as the electron mass, and of course the neutrino masses, if any. This mechanism, named by some after a children's playground device, relates smallness to the ratio of scales. The only interesting question is the origin of the $\Delta I_w = 0$ scale.

ii) Spinless Extensions

Without new neutral fermions, the mass matrix for neutrinos is of Majorana type, and violates lepton number. The only freedom therefore is to add Higgs fields which, unlike the Higgs of the Standard Model, carry lepton number, which the new Higgs acquire through their Yukawa couplings to the leptons. If the new Higgs have no color, they can couple to the following combinations

$$L^T_{(a}\sigma_2\tau_2\tau L_{b)} \sim (1, 1; 3, 1^c)_{-2},$$
$$L^T_{[a}\sigma_2\tau_2 L_{n]} \sim (1, 1; 1, 1^c)_{-2},$$
$$\bar{e}_{(a}\sigma_2\bar{e}_{b)} \sim (1, 1; 1, 1^c)_4 .$$

This yields new Higgs with two units of lepton number, an isotriplet, and two isosinglets, one singly charged, the doubly charged. These couplings do not violate lepton number, they serve to define the Higgs lepton number. The new Higgs in turn are now free to interact in such a way that lepton number is broken, either explicitly by some term in the potential [9], or spontaneously, which can happen for the triplet. Then we know that lepton number violation by these new terms opens the way for the dimension five operators to give a radiative Majorana mass to the neutrinos, providing an automatic suppression of the mass. Of all the Higgs which acquire lepton number by Yukawa coupling the the Standard Model, only the weak isotriplet has a neutral component. If the potential is such that this component gets a vacuum value, one of two things can happen: either the potential preserves lepton number, or it breaks it explicitly. In the former case, the vacuum value breaks spontaneously lepton number. In this minimal extension, there results a Nambu-Goldstone boson, called the (triplet) Majoron [10]. This triplet Majoron model is already ruled out by experiment through its coupling to the Z boson. In the latter case, there is no Nambu-Goldstone boson, and the only puzzle is the smallness of the neutrino masses; one has to device a model where the vev of the triplet is naturally much smaller than those of the doublets.

In the other cases with no extra Higgs other than those just listed, and where there are no tree level Majorana masses, one has to rely on explicit breaking of lepton number to generate Majorana masses. This is problematic for lepton number, because it severely limits possible mechanisms for generating baryon asymmetry. The culprit is the KRS effect [11] which links baryon number to lepton number in the early universe. If L is not a symmetry of the Lagrangean, there is no place for baryon number to escape the sphaleron, and we are left at the mercy of mechanisms which generate baryon asymmetry at yet lower temperatures. Thus it would be preferable to avoid explicit breaking of lepton number.

If L is to be broken explicitly, we can distinguish between models by the electroweak quantum numbers of the lepton number carrying field that get a vev. It is not the purpose of these lectures to go in the building of Majoron models. It suffices to mention that the simplest doublet Majoron models also conflict with the Z-width. The simplest model which still survives experiment is the singlet Majoron model [12] where the field with a vev carries only lepton number. In general, a Nambu-Goldtone boson couples through the divergence of the current that generates the spontaneously broken symmetry. The coupling strength inversely proportional to the vev. The limits on this vev are obtained by the usual stability argument of old stars.

In GUT models beyond the simplest $SU(5)$, lepton number is part of a gauged symmetry, and the Majoron is eaten by a massive vector.

4. Neutrino Oscillations

Back in 1957 [13], motivated by rumors of evidence that neutrinos were sometimes produced in beta decay, Pontecorvo [14] proposed that one should consider, in analogy with the $K_0 - \bar{K}_0$ system, that antineutrinos produced in normal beta decay might oscillate into neutrinos. The rumors turned out to be just that [15], but the idea of neutrino oscillations remained. This suggestion was made before it was known that there were several kinds of neutrinos [16]. That year, the group at Nagoya [17], proposed the idea of oscillations with an arbitrary mixing angle between neutrinos species, which we now call flavor oscillations.

We now discuss flavor oscillations among two neutrinos, which preserve total lepton number. The case of neutrino-antineutrino oscillations which breaks lepton number is much more difficult to track since its effects would necessarily be suppressed by the ratio of the small Majorana mass to a large momentum. In vacuum, the (hatted) neutrino mass eigenstates satisfy the equation

$$\frac{d}{dt}\begin{pmatrix}\hat{\nu}_e\\ \hat{\nu}_\mu\end{pmatrix}=\begin{pmatrix}E_e & 0\\ 0 & E_\mu\end{pmatrix}\begin{pmatrix}\hat{\nu}_e\\ \hat{\nu}_\mu\end{pmatrix},$$

where the energies are given by

$$E_e=\sqrt{p^2+m_1^2}\approx p+\frac{m_1^2}{2p},$$

$$E_\mu=\sqrt{p^2+m_2^2}\approx p+\frac{m_2^2}{2p},$$

The mass eigenstates $\hat{\nu}_e(t)$ and $\hat{\nu}_\mu(t)$ are related to the unhatted **weak** eigenstates through

$$\begin{pmatrix}\nu_e(0)\\ \hat{\nu}_\mu(0)\end{pmatrix}=\begin{pmatrix}\cos\theta_0 & \sin\theta_0\\ -\sin\theta_0 & \cos\theta_0\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix},$$

where θ_0 is the vacuum mixing angle.

We can then rewrite the evolution equation in the form

$$i\frac{d}{dt}\begin{pmatrix}\hat{\nu}_e\\ \hat{\nu}_\mu\end{pmatrix}=\frac{1}{2}(E_e+E_\mu)\begin{pmatrix}\hat{\nu}_e\\ \hat{\nu}_\mu\end{pmatrix}+\begin{pmatrix}\frac{2\pi}{L_0} & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\hat{\nu}_e\\ \hat{\nu}_\mu\end{pmatrix}.$$

The parameter L_0 with dimension of length is given by

$$L_0=\frac{4\pi p}{(\Delta m^2)}.$$

The unit diagonal component of the mass matrix will play no role in oscillations which depend only on differences between the two states.

At $t=0$ an electron neutrino state is created in a weak interaction; it is given by a linear combination of mass eigenstates

$$|\nu_e\rangle=\cos\theta_0|\hat{\nu}_e\rangle+\sin\theta_0|\hat{\nu}_\mu\rangle.$$

At a later time t, that neutrino state will look like

$$|\nu_e(t)\rangle=\cos\theta_0 e^{-iE_e t}|\hat{\nu}_b\rangle+\sin\theta_0 e^{-iE_\mu t}|\hat{\nu}_\mu\rangle.$$

Suppose that we use a detector which triggers on the weak eigenstates ν_μ. The (appearance) probability of triggering that detector at the time t will be

$$P_{(e\to\mu)}(t)=|\langle\nu_e(t)|\nu_e(0)\rangle|^2,$$
$$=\frac{1}{2}\sin^2 2\theta_0\sin^2\frac{2\pi t}{L_0}.$$

The probability peaks whenever the neutrino has travelled a distance equal to one quarter of L_0. In this case we have computed the appearance probability; it is proportional to the square of the mixing angle. The distance between peaks is L_0, the oscillation length. A convenient way to remember the size of the oscillation length is

$$L_0=2.47\frac{E(\text{GeV})}{\Delta m^2(\text{eV}^2)}\text{ meters}.$$

Many types of experiments have searched for manifestations of neutrino masses [18]. One type is kinematic searches, which has yielded the remarkable bound of 9.2 eV for the electron neutrino. Another has been oscillations. So far the result of laboratory oscillations have been negative, ruling out certain regions of parameter space in the $\Delta m^2-\sin^2 2\theta_0$ plane. These experiments fall into several categories. In the first one monitors the antineutrinos coming from reactors. Life is made difficult by the fact that the antineutrinos are not directional. Thus the flux falls sharply with distance. In the second class one monitors neutrino and antineutrino beams at High Energy Accelerators. Recently, however, some hope has been offered in the monitoring of solar neutrinos, where there is an apparent deficit. At the time of this writing, it is not known whether the reasons behind this effect is to be found in our imperfect understanding of solar models, in our ignorance of the detector efficiencies, or else in the manifestation of fundamental physics: neutrino oscillations. Two new detectors capable of deciding the solar model issue are coming on line; alas neither is calibrated, but one has produced some intriguing preliminary data.

5. Solar Neutrinos: Production and Detection

According to the theory of the solar engine [19], electron neutrinos are produced during the so-called $p-p$ chain which starts with protons and ends up with α particles. Neutrinos are also produced during the CNO cycle as well, but in such reduced rates that we do not discuss the CNO neutrinos. In fact the $p-p$ chain is responsible for 98.5% of the energy generated by the Sun. The $p-p$ chain reactions are as follows:

$$p+p\to{}^2H+e^++\nu_e,\quad 0<E_\nu<0.420\text{ MeV},$$
$$p+e^-+p\to{}^2H+\nu_e,\quad E_\nu=1.44\text{ MeV},$$
$${}^2H+p\to{}^3He+\gamma,$$
$${}^3He+{}^3He\to{}^4He+p+p,$$
$${}^3He+{}^4He\to{}^7Be+p,$$
$${}^7Be+e^-\to{}^7Li+(\gamma)+\nu_e,\quad E_\nu=\begin{cases}0.861\text{ MeV }(90\%)\\ 0.383\text{ MeV }(10\%)\end{cases},$$

$$^{7}Be+p \rightarrow {}^{8}B+\gamma\,,$$
$$^{8}B \rightarrow {}^{8}Be+e^{+}+\nu_e\,, \quad 0.81 < E_\nu < 14.06 \text{ MeV},$$
$$^{7}Li+p \rightarrow {}^{4}He+{}^{4}He\,,$$
$$^{8}Be \rightarrow {}^{4}He+{}^{4}He\,.$$

The reason for the two lines in ^{7}Be capture is that 10% of the time the ^{7}Li is produced in a metastable state which then radiatively decays. Thus we see that we have three neutrino lines, and two continuous bands, one at low energy coming from the original fusion reaction, the other at higher energies coming from the decay of Boron. The flux of the lower energy neutrinos depends on the proton density while the flux of the higher energy neutrinos depends on the abundance of ^{8}B. Clearly the amount of Boron is much more environmentally sensitive than that of the primeval protons. This translates into the dependence of the neutrino flux on the Sun's temperature. The flux of neutrinos from Boron decay depend on the 18th power of the Sun's temperature, to be compared with those from the fusion reaction which depend only on the (-1.2)th power of temperature. Thus particle physicists may be excused if they do not attribute a deficiency in the neutrino flux from Boron decay to a small deviation of the temperature from its accepted value. However, a similar deviation from the expected solar model from the fusion neutrinos would point at causes other than the solar model. The detection of these neutrinos on this planet proceeds in three types of experiments.

- The first experiment, performed deep underground in the Homestake mine [20], relies on the reaction

$$\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar\,.$$

Energetics is such that only neutrinos with at least 0.814 MeV energy are capable of producing Argon. Only neutrinos coming from the $p-e-p$ capture, Beryllium capture and the Boron decay can trigger this detector. This experiment has been performed for over twenty years albeit with funding interruptions.

- The second experiment [21] is of the same type, but uses cleverly chosen elements to be triggered by much lower energy neutrinos. The reaction is in this case

$$\nu_e + {}^{71}Ga \rightarrow e^- + {}^{71}Ge\,.$$

It can be triggered by neutrinos with as little energy as 0.23 MeV. Thus it is sensitive to the neutrinos that come from the primeval fusion reaction, as well of course as those that come from all the other sources. However, the flux of the $p-p$ neutrinos is many orders of magnitude higher than the flux emanating from Boron decay. This type of experiment is now being conducted at two underground detectors, SAGE [21] near Baksan in the Caucasus mountain range, the other GALLEX in the Gran Sasso tunnel near Rome.

- The third experiment is conducted in Japan at the Kamiokande [22] (proton decay) detector. There a neutrino entering the detector can scatter with an electron

$$\nu_e + e^- \rightarrow \nu_e + e^-\,.$$

The electron recoils in the direction it was hit. By detecting the recoil electron, one should find an excess of electrons opposite the direction of the Sun at the time of impact. This detector can only detect recoil electrons if they have been hit sufficiently hard, limiting the detection to neutrinos with an energy greater than 7.3 MeV, at the upper end of the Boron decay. At this time, the results from these detectors are as follows, in the convenient Solar Neutrino Unit (SNU), which equal 10^{-36} captures per target atom per second.

Experiment	Predicted (SNU)	Observed (SNU)
^{37}Cl	6–9	2.1 ± 0.3
SAGE ^{71}Ga	132	<80**
GALLEX ^{71}Ga	132	No result**
Kamiokande	1*	0.39 ± 0.13

* Arbitrary normalization, ** Since the miting of this article in 1990, both Gallium detectors, SAGE and GALLEX, have been calibrated; they observe

SAGE	74	$^{+13+5}_{-12-7}$	SNUs
GALLEX	79	$\pm 10 \pm 6$	SNUs

to be compared with the predicted number ~ 132 SNUs.

At face value, there seems to be a deficit by a factor of two in the two oldest experiments. The preliminary data from the Gallium experiment is to be viewed as preliminary. A sage person should wait before beating the drums and getting all excited. Still in this time of novel data drought, one should be forgiven some excitement [23]. The interesting thing is that both Homestake and Kamiokande see a deficit. As of this writing, the Gallium experiments are running, so that we may expect in a year's time some results, which could settle the uncertainty associated with solar models. However, one must remember that neither detector is calibrated, although efforts are being made in that direction: neutrino sources do not come easily; they have to be manufactured, and then transported to the site before they decay completely. The future of solar neutrino detection is bright. New experiments are being planned in the Gran Sasso tunnel, BOREX, BOREXINO. Also a deuterium detector called SNO will come on line within the decade and it will have the capacity of detecting flavor independent neutral current effects.

6. Solar Neutrino Oscillations

The most outrageous possibility, considered long ago [24], is that the Earth is at the right distance from the Sun to be in a through in the variation of the probability with distance (even massive neutrinos would travel at nearly the speed of light). The Earth–Sun distance is 1.5×10^{11} meters. This would happen if $\Delta m^2 \approx 10^{-11}\ \mathrm{eV}^2$! With precise data on pp neutrinos, which have a much smaller oscillation length than the Chlorine neutrinos, it will be possible to decide on this hypothesis.

Solar neutrinos are emitted at or near the core of the Sun: in order to get to us they must traverse the Sun. Wolfenstein [25] noted that because of coherent forward scattering, the Sun's interior acts as a refractive medium to neutrinos, and that different species will have different indices of refraction. The combination of these two effects, oscillations due to vacuum mixiing and the different indices of refraction, can lead [26] to the vanishing of the diagonal element of the effective mixing matrix, and thus produce maximum mixing for a large range of the fundamental parameters. This is the MSW effect, which we now describe.

Consider [27] a wave traversing a slab of matter of length R with N scattering centers per unit area. The wave function will be the sum of the incoming and scattered waves,

$$\Psi(x) = \Psi_{\mathrm{in}}(x) + \Psi_{\mathrm{scat}}(x)\,,$$

which in lowest order of scattering yields

$$\psi(x) \approx \exp\left[ipx + \frac{2\pi i N R}{p} f_p(0)\right],$$

where $f_p(0)$ is the forward scattering amplitude. We then rewrite the total wave function as what it would have been without the slab multiplied by a modulating factor

$$\Psi(x) \approx e^{ip(x-R)} e^{ipRn(p)}\,,$$

where $n(p)$ is the index of refraction

$$n(p) = 1 + \frac{2\pi}{p^2} N f_p(0)\,.$$

The imaginary part of $f_p(0)$ parametrizes the extinction (optical theorem), and the real part of $f_p(0)$ just changes the propagation properties of the plane wave. An electron neutrino will interact with the electrons in the Sun through elastic forward scattering, through W and Z mediated processes, as well as with neutrons and protons through Z exchange. Thus we have for the electron neutrino weak eigenstate an index n_e given by

$$p(n_e - 1) = \frac{2\pi}{p}\left[N_e f_p^{\mathrm{W}}(0) + \sum_i N_i f_p^{\mathrm{Z}}(0)\right],$$

where the W and Z contributions have been singled out. A muon neutrino weak eigenstate, on the other hand, will not have any W mediated interactions since there are no muons in the Sun. However, its neutral current interactions will be the same, leading to its index of refraction n_μ,

$$p(n_\mu - 1) = \frac{2\pi}{p}\left[0 + \sum_i N_i f_p^{\mathrm{Z}}(0)\right].$$

For a sterile neutrino which has no interactions with matter, the index of refraction n_s will just be one,

$$p(n_s - 1) = 0\,.$$

These three types of neutrinos propagate through matter quite differently. For the weak eigenstates,

$$|\nu_e(x)\rangle = e^{ipn_e x}|\nu_e\rangle\,,$$
$$|\nu_\mu(x)\rangle = e^{ipn_\mu x}|\nu_\mu\rangle\,,$$
$$|\nu_s(x)\rangle = e^{ipx}|\nu_s\rangle\,.$$

The real part of the index of refraction leads to a change in potential energy proportional to $p(n-1)$; it contributes to the mixing matrix in the mass eigenstates basis

$$\begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix}\begin{pmatrix} p(n_e-1) & 0 \\ 0 & p(n_2-1) \end{pmatrix} \cdot \begin{pmatrix} \cos\theta_0 & -\sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{pmatrix},$$

which is to be added to the already diagonal energy matrix. We discard all the irrelevant elements proportional to the unit matrix, define another length L_m by the relation

$$\frac{2\pi}{L_m} \equiv p(n_1 - n_2),$$

as well as the ratio

$$\chi \equiv \frac{L_0}{L_m},$$

and rewrite the mixing matrix in the mass eigenstate basis

$$\frac{2\pi}{L_0}\begin{pmatrix} 1+\chi\cos 2\theta_0 & -\chi/2\sin 2\theta_0 \\ -\chi/2\sin 2\theta_0 & 0 \end{pmatrix},$$

or in the weak eigenstate basis as

$$\frac{2\pi}{L_m}\begin{pmatrix} 1+1/\chi\cos 2\theta_0 & 1/2\,\chi\sin 2\theta_0 \\ 1/2\,\chi\sin 2\theta_0 & 0 \end{pmatrix}.$$

We proceed to diagonalize this matrix by means of an orthogonal rotation with angle θ_m, where elementary algebra gives us

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta_0}{(1+2\chi\cos 2\theta_0 + \chi^2)}.$$

The variable of interest, χ, depends on the species of neutrinos considered. For the case of $\nu_e - \nu_\mu$ flavor oscillations, the contributions to L_m from the neutral current cancel out and we are left with the contribution from charged current interactions with electrons.

In the Standard Model, the ratio χ is given by

$$\chi = 2\sqrt{2}\,G_F\,p\,\frac{N_e}{\Delta m^2},$$

where G_F is the Fermi constant, N_e is the local electron density, and Δm^2 is the square of the difference of masses. It is negative if the first species is lighter than the second. Outside a central core region, the electron density inside the Sun decreases exponentially

$$N_e(r) = N_{eC}\,e^{-\frac{r}{r_0}},$$

where N_{eC} is the density at the core region, roughly one hundred Avogadro's number at one tenth the radius, and r_0 is itself about one tenth the Sun's radius $r_0 \approx 7 \times 10^7$ meters. From the formula for the matter mixing matrix, we see that the MSW resonance occurs whenever

$$\chi = -\frac{1}{\cos 2\theta_0},$$

at which point the diagonal element vanishes and the off-diagonal elements, no matter how small, produce maximum mixing. It corresponds to a critical electron density

$$N_e^{\text{crit}} = -\frac{\Delta m^2\cos 2\theta_0}{2\sqrt{2}\,G_F E_\nu}.$$

Let us define the region of resonance as that over which $\sin^2 2\theta_m \geq \frac{1}{2}$. Over that region χ varies according to $\Delta\chi = \sin 2\theta_0$, corresponding to a variation of the density

$$\Delta N_e = \frac{\Delta m^2\cos 2\theta_0}{2\sqrt{2}\,G_F E_\nu}.$$

In that region, the mixing matrix looks like

$$\frac{\pi}{L_0}\begin{pmatrix} 0 & \sin 2\omega_0 \\ \sin 2\theta_0 & 0 \end{pmatrix},$$

so that the oscillation length at enhancement is given by

$$L_{\text{enh}} = \frac{4\pi E_\nu}{\Delta m^2 \sin 2\theta_0}.$$

Physically, we can envisage two extreme possibilities. In the first "adiabatic" [28] approximation, the region over which the enhancement takes place is much larger than the oscillation length. The other is the "slab" [29], or "non-adiabatic" regime where the oscillation length is much longer than the resonance region, Δr.

$$L_{\text{enh.}} \ll \Delta r: \text{ Adiabatic Regime,}$$
$$L_{\text{enh.}} \gg \Delta r: \text{ Non-Adiabatic Regime.}$$

Which of these obtains depends of course on the value of the fundamental parameters. Qualitatively, we expect the size of the enhancement region to depend on where it lies inside the Sun. If it is in the core region where the electron density does not change appreciably, the size Δr can be expected to be large. On the other hand, if it occurs outside the core region, we expect the size to be much smaller, namely

$$\Delta r = 2\,r_0 \tan 2\theta_0.$$

We therefore expect the slab regime to apply at the periphery, if at all, and the "adiabatic" cas to apply for enhancement near the Sun's center. Note that if the MSW effect is to occur at all, the electron density must be such that the condition for the critical density is met. Since N_e is always less than its value N_{eC} at the core, this gives a critical value for the neutrino energy

$$E_\nu^{\min} = -\frac{\Delta m^2 \cos 2\theta_0}{2\sqrt{2}\, G_F N_{eC}}.$$
$$= 4 \times 10^4 \,|\Delta m^2|\, (\mathrm{eV}^2) \cos 2\theta_0 \text{ in MeV}.$$

Neutrinos with energy less than $E_\nu^{\min}$ do not encounter the resonance layer. Since the detectable solar neutrinos range in energy from 0.23 MeV to 14 MeV, this limits the range of fundamental parameters for which this effect is measurable.

Let us first consider the adiabatic case. In this case, neutrinos which go through many oscillations before they leave the resonance region; those with enough energy will be totally converted into other neutrinos, undetectable by present detectors. If the energy threshold for the effect lies in the range of the solar neutrino detectors, one expects a more significant reduction in the experiment with the highest threshold.

In the non-adiabatic "slab" case, the resonance region is much smaller than the oscillation length. This means that only part of the wave gets lost through oscillation. One can compute the probability that an electron neutrino makes it through the slab without a personality change. In the slab, the probability undergoes oscillatory behavior since the diagonal element of the mixing matrix vanishes there. This is the same as for the adiabatic case, but here only a fraction of an oscillation takes place before the neutrino leaves the slab, so that the probability amplitude will keep its oscillatory character. The survival probability is

$$P_{\nu_e \to \nu_e} \equiv \cos^2\left(\frac{\Delta m^2 \cos 2\theta_0}{2 r_0 E_\nu}\right);$$

it is dependent on the energy of the neutrino. In the non-adiabatic case, even if it has energy above threshold, a neutrino will not convert if its energy is large enough. Hence a possible scenario emerges: the *pp* neutrinos which have lower energy will be depleted, and by the time the energy has risen to the Homestake and Kamiokande thresholds, the probability of detection has gone through its cycle and is rising. We expect from the possibility much more depletion at SAGE than is seen at Homestake and Kamiokande. One can check that this case obtains when the slab is at the Sun's periphery. For enhancement outside the core, the adiabaticity ratio is given by

$$\varepsilon = 1.79 \times 10^{-8} \frac{E_\nu(\mathrm{MeV})}{\Delta m^2(\mathrm{eV}^2)} \frac{\cos 2\theta_0}{\sin^2 2\theta_0},$$

which tells us the relevant value of the parameters. We also note that it can occur for much lower energy neutrinos since the critical electron density is much lower, by several orders of magnitude over its value at the core.

Which of these regimes is favored by experiment? This depends very much on which experiment one wishes to take seriously. At present the Homestake and Kamiokande results are more credible, one because of its longevity and consistency, the other because of the simplicity of the detector. The qualitative difference between the two detectors is that Kamiokande triggers on higher energy neutrinos, while the Homestake experiment triggers on *pep*, *Be* capture and CNO cycle neutrinos. Yet the observed suppression from Solar theory is the same for both, so that there does not seem to be much of an energy dependence for the higher energy neutrinos. Suppose the MSW region is at the Sun's center. The threshold for escaping detection is

$$E_\nu(\mathrm{MeV}) \geq \Delta m^2(\mathrm{eV}^2)\, 4 \times 10^4 \cos 2\theta_0 .$$

Since both Homestake and Kamiokande do detect Solar neutrinos, this would imply that this critical threshold should be above Kamiokande's threshold of 7.3 MeV, say around ten MeV's so that

$$\Delta m^2 \cos 2\theta_0 \approx 2 \times 10^{-4}\, \mathrm{eV}^2 .$$

As long as the vacuum angle is not close to $\pi/2$ a very specific prediction emerges. However, for this picture to be consistent, all the neutrinos coming from the *pep*, *Be* capture, CNO cycle, and Boron decay with energy less than ten MeV's would be detected by Homestake, accounting for twice as many as observed, ruling out the effect. This analysis [30] relies on almost religious belief in both the Standard Solar Model, and the data from these two detectors. In addition, in this approximation, the SAGE and GALLEX detectors should observe the predicted number of neutrinos, since none of the *pp* neutrinos would convert.

On the other hand, if the MSW region is at the Sun's periphery, we can expect a depletion from the *pp* neutrinos. The MSW effect can occur for much lower values of the mass difference, since the electron density is so much smaller. In fact, the domain of applicability of the effect, assuming that the Homestake and Kamiokande values are correct, is given by [29]

$$|\Delta m^2| \sin^2 2\theta_0 \approx 10^{-7.5}\, \mathrm{eV}^2.$$

It is exciting that this range of values falls in one of the theoretical prejudice bins. However, one must beware of sociological fixed points: mere belief does not make it true. That is the reason we must eagerly wait for the results from the Gallium detectors.

7. Conclusions

As in George's time, neutrinos are proving to be very powerful probes for determining the structure of fundamental interactions. We have not discussed the many other roles that neutrinos are thought to play, such as Dark Matter candidates, or seeds in generating large scale structure. There are many hints coming from many diverse experiments that neutrinos are indeed massive. While none are conclusive, the existence of neutrino masses is of monumental import as it would force us to expand our horizons beyond the Standard Model. Just as the V–A formulation led us in short time to the Standard Model, we hope that data involving neutrinos will lead us once again to a more concise formulation of the fundamental interactions.

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